'FLUIDS'

State of matter wich can flow are called fluid.

Therefore, liquid & gases are in the category of fluids. They do not have any shape & size their own but have capability to Aquire the shape of container.

Types of fluid →

→ Ideal fluid are those wich are incompressible i.e volume fixed. * They have property of zero Resistance i.e viscosity & surface tenson assumed to zero.

Density (ρ) : (Mass density (ρ)) →

For uniformally distributed mass, ρ is defined as Mass/volume

$$\rho = \frac{\text{Mass}}{\text{volume}} = \frac{\Delta m}{\Delta V}$$

But, incase of non-uniformally distributed mass, density at a point is given by -

$$\rho = \frac{dm}{dV}$$

$$\rho_{Water} = 1\,g/cc = 1000\,kg/m^3$$

Weight density (ω)

$$W = \frac{(\Delta m)g}{\Delta V}$$

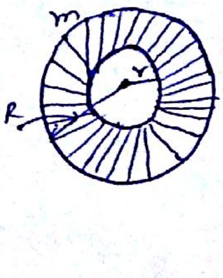
$$\omega = \rho g$$

Relative density

$$= \frac{\text{density of substance}}{\text{density of water at } 4^\circ C}$$

↳ $1\,g/cm^3 = 10^3\,kg/m^3$

*** # 'ρ' of Material & 'ρ' of body →



$$\rho_{body} = \frac{m}{\frac{4}{3}\pi R^3}$$

$$\rho_{Material} = \frac{m}{\frac{4}{3}\pi (R^3 - r^3)}$$

* Both have equal density If there in cavity.

$$\rho_{body} \leqslant \rho_{Material}$$

Density of Mixture :→

*→ Two liquid of density ρ_1 & ρ_2 are mixed, find density of mixture, If they are

(i) → Mixed in equal mass → $\rho = \dfrac{2\rho_1\rho_2}{\rho_1+\rho_2}$ (H·M)

(ii) → Mixed in equal volume → $\rho = \dfrac{\rho_1+\rho_2}{2}$ (A·M)

* $\rho = \dfrac{m_1+m_2}{V_1+V_2}$

$$\rho = \frac{m_1+m_2}{\frac{m_1}{\rho_1}+\frac{m_2}{\rho_2}}$$

Effect of temp.

$$\rho_0 = \rho(1+\gamma\Delta\theta)$$

* $$\rho = \frac{\rho_0}{1+\gamma\Delta\theta}$$

γ → volume cofficient of expansion

V_0 → Initial volume.

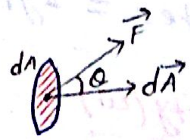
ρ_0 → Initial density.

Pressure (P) :→

* If force is uniformally distributed over an area then, $P = \dfrac{F_N}{A} = \dfrac{\Delta F_N}{\Delta A}$

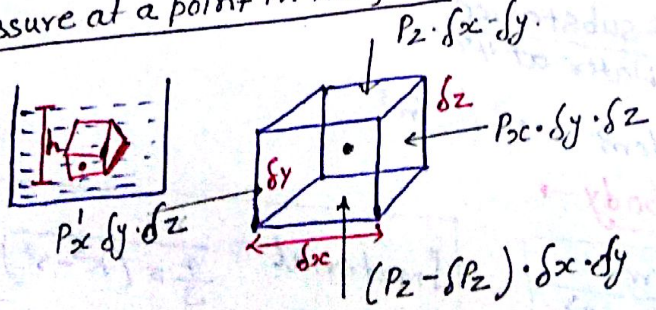
* If force is not uniformally distributed over the area, then pressure at a point defined as,

$$P = \frac{dF_N}{dA} = \frac{\vec{F}\cdot d\hat{A}}{dA}$$



* It is scalar quantity.

* Pressure at a point in the fluid →



As fluid is at Rest :→

$P_x = P'_x \Rightarrow \delta P_x = 0$

Similarly, $P_y = P'_y \Rightarrow \delta P_y = 0$

$$-\frac{\delta P_z}{\delta z} = \rho g$$

$$\therefore \boxed{-\frac{dp}{dz} = \rho g}$$

or, $-\vec{\nabla} \cdot p = \rho g \vec{k}$ in ⊕ve z-direction (vertically upward),

* ⊖ve sign indicate that pressure is decresing.

$$\therefore p = \rho g h + c$$

* At $h = 0$, $p = p_0$

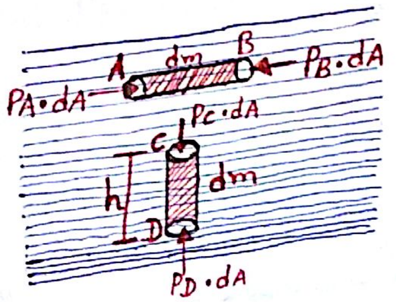
$$\boxed{p = p_0 + \rho g h}$$

$\rho g h \Rightarrow$ gauge pressure.

h = vertical depth from the free surface.

p = absalute pressure.

#



*For AB

As fluid is at Rest

$\therefore$ net force of dm is zero.

$$\therefore P_A \cdot dA = P_B \cdot dA$$

$$\therefore \boxed{P_A = P_B}$$

*For CD

As fluid is at Rest $\therefore$ net force on 'dm' is zero.

$$P_D \cdot dA = P_C \cdot dA + \rho \cdot (dA) h g$$

$$* \boxed{P_D = P_C + \rho g h}$$

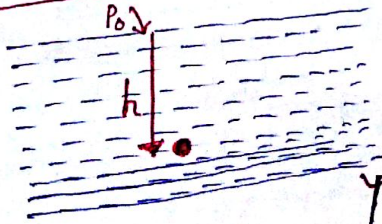
$\rightarrow \boxed{\vec{\nabla} \cdot p = -\rho g \vec{k}}$

$\rightarrow \boxed{\frac{dp}{dz} = -\rho g}$

$\rightarrow$ If 'h' depth $\therefore z = -h$

$$\boxed{\frac{dp}{dh} = \rho g}$$

Free surface (Atmosphere) -



as $\frac{dp}{dt} = \rho g$

$p = \rho g h + c$

at $h = 0, p = p_0$

$\therefore c = p_0$

$$\boxed{p = p_0 + \rho g h}$$

NOTE

* In case of stationary fluid; pressure at all the points lying on same horizontal plane are equal.
* At a point inside liquid, pressure is Isotropic. i.e in every direction the effect of pressure is same at a point.

* Pressure at a point inside liquid at depth 'h' from the free surface is

$$P = P_0 + \rho g h$$

Where, 'P' is absolute pressure at that point & P_0 is atmospheric pressure. '$\rho g h$' is due to weight of liquid & called 'guage pressure.'

P = absolute pressure (Always +ve)

Guage pressure = $\rho g h = P - P_0$ [may be +ve, −ve or zero]

Bulk Modulus $B = \frac{V_0 \Delta P}{\Delta V}$

Initial density $\rho = \frac{m}{V_0}$

$$\rho = \frac{\rho_0}{1 + \frac{\Delta P}{B}}$$

(value put With sign)

* $$\rho = \rho_0 \left(1 - \frac{\Delta P}{B}\right)$$

(ΔP singn के साथ होगा)

S.I unit → N/m^2 (pascal)

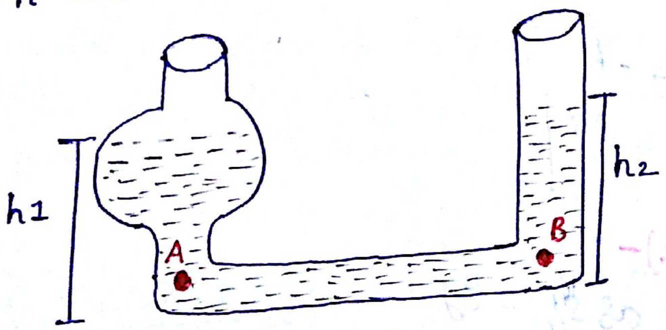
$$1 \, Pa = 1 \, N/m^2$$

Barr, torr is also unit of press.

[1 Barr = 10^5 pa
1 torr = 133 Pa]

Hhydrostatic paradox

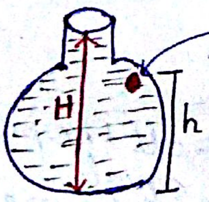
Pressure at a point inside liquid, depends only on $P_0, \rho, g,$ h not on the shape & size of liquid column.



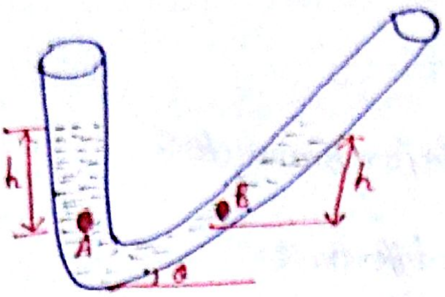
$P_A = P_B$

$P_0 + \rho g h_1 = P_0 + \rho g h_2$

$$h_1 = h_2$$



$$P = P_0 + \rho g (H - h)$$



$P_A = P_B$

$P_0 + \rho g h = P_B$

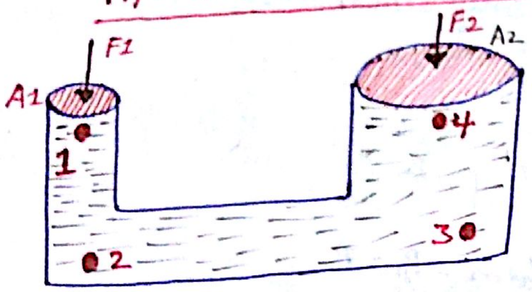
$\therefore P_B = P_0 + \rho g h$

$= \boxed{P_0 + \rho g (y \sin\theta)}$

* Pressure exerts on wall of container always perpendicular to surface of wall.

Whenever pressure at a point in a stationary fluid is changed, then this change distributed to every part of fluid without diminishing.

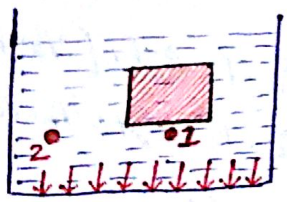
Hydrolic lift or, Hydrolic Break



$\boxed{\frac{F_1}{A_1} = \frac{F_2}{A_2}}$

$\boxed{A_2 > A_1 \quad F_2 > F_1}$

*

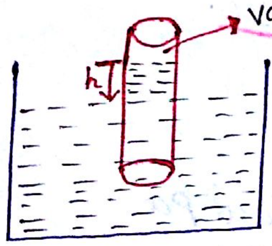


$\boxed{P_1 = P_2}$

* If a point is open to atmosphere, then pressure at that point will always be equal to atmospheric pressure.

Barometer

It Reads atmospheric pressure.



* h also known as Pressure head.

$P_0 = 0 + \rho g h$

$P_0 = \rho g h$

* For Barometric liquid → Hg.

then, $1\ atm = 1.01 \times 10^5\ Pa$

$1.1 \times 10^5 = \rho_{Hg} \cdot g \cdot h$

$\boxed{\rho_{Hg} = 13.6\ g/cm^3 = 13.6 \times 10^3\ kg/m^3}$

$1.01 \times 10^5 = 13.6 \times 10^3 \times 10 \times h \Rightarrow h = \frac{10.1}{13.6} = 0.76 m$

$\boxed{h = 0.76 m = 76 cm = 760 mm}$

$\therefore \boxed{1\ atm = 760\ mm\ of\ Hg}$

$\boxed{1\ atm = 1.03 \times 10^5\ N/m^2}$

* For Barometric liquid, Water.

then, at 1 atm $= 1.01 \times 10^5$ Pa

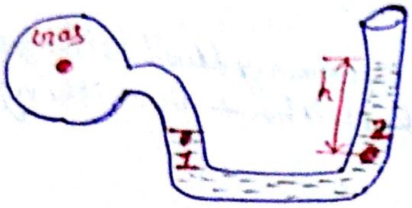
$1.01 \times 10^5 = 10^3 \times 10 \times h$

$h = 10.1$ m

So, Water is not suitable for Barometer.

Simple Manometer or, U-tube Manometer

It Reads guage pressure or, Pressure difference.



$P_1 = P_2$

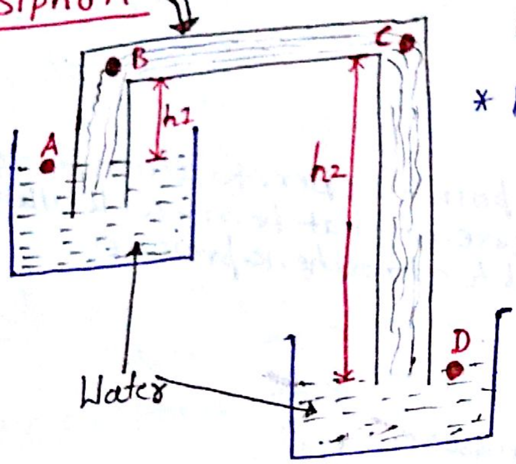
$P_g = P_0 + \rho g h$

$P_{gauge} = P_g - P_0$

$P_{gauge} = \rho g h$

NOTE→ Pressure at two points on the same horizontal plane, lying in stationary fluids will be equal only When these points lie in the same liquid.

Siphon



Find P_A, P_B, P_C, P_D

* $P_A = P_{atm} = P_D = P_0$

$P_B \neq P_C$

$P_A = P_B + \rho g h_1 \Rightarrow P_B = P_0 - \rho g h_1 \; [\because P_A = P_0]$

$P_D = P_C + \rho g h_2 \Rightarrow P_C = P_0 - \rho g h_2 \; [P_D = P_0]$

$P_B - P_C = \rho g (h_2 - h_1)$

***→ If siphon is Working, then there should not be vaccume at any point, hence $P_B > 0$.

$P_0 - \rho g h_1 > 0$

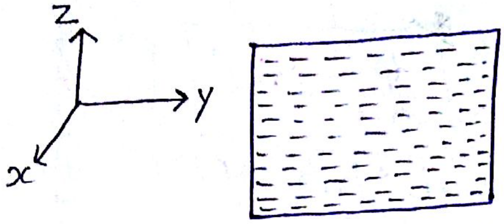
$h_1 < \frac{P_0}{\rho g}$

If $P_0 = 1$ atm $= 1.01 \times 10^5$ Pa

* $h_1 < 10.1$ m

Pascal law → When liquid is filled in a container & are in equilibrium & When press. is applied at any part then transmited to every part of liquid Without loss.

Accelerated fluids



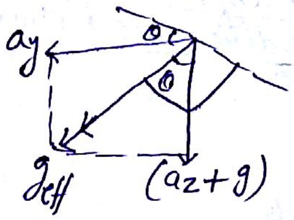
$\vec{a}_c = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\vec{\nabla} \cdot P = -\rho[a_x\hat{i} + a_y\hat{j} + (a_z+g)\hat{k}]$

$\vec{\nabla} P = -\rho[a_x\hat{i} + a_y\hat{j} + (a_z+g)\hat{k}]$

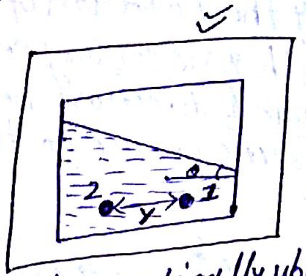
Exemplar **Effective Gravity Method**

NOTE → The free surface of liq. is perpendicular to effective gravity.



IIT

* $$\tan\theta = \frac{a_y}{a_z+g}$$



Modified Pascal's Law →

If x–y is horizontal plane & (+)ve z is vertically upward,

then If, a_c = accln of container $= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\vec{a}_c = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$$\frac{\partial P}{\partial x} = -\rho a_x$$

$$\frac{\partial P}{\partial y} = -\rho a_y$$

$$\frac{\partial P}{\partial z} = -\rho(a_z+g)$$

$$\therefore \vec{\nabla} \cdot P = -\rho[a_x\hat{i} + a_y\hat{j} + (a_z+g)\hat{k}]$$

If $a_x = 0$, then,

(i) → P increase linearly along −z axis;

According to $\boxed{P_2 = P_1 + \rho(g+a_z)h}$

point ② is below point ①. Where h = depth of ② below ①

(ii) → P ↑ is linearly along −y axis according to

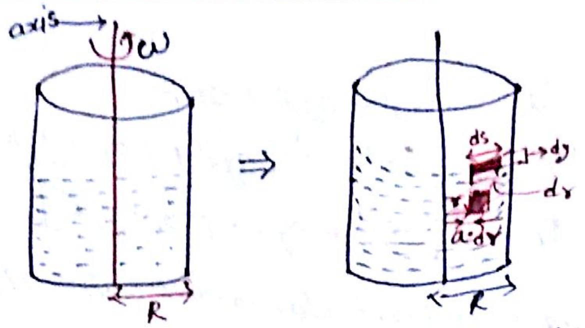
$\boxed{P_2 = P_1 + \rho a_y \cdot y}$

Where, y = distance b/w point ① & ② along y-axis

(iii) → If Inclination of free surface with horizontal 'θ' then, $\boxed{\tan\theta = \frac{a_y}{a_z+g}}$

(iv) → use effective gravity for shortcut & Remember that free surface is always perpendicular to effective gravity.

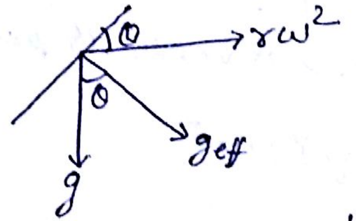
Rotation of Liquid container →



$$\boxed{\frac{dp}{dr} = \rho r \omega^2}$$

with ↑ in 'r' ⇒ P↑

*** By the Rotation of container, all liquid particle move in a circular path & necessory centripetal force is provided by pressure difference along radial direction, hence there is a pressure difference in horizontal direction, due to this Rotation.

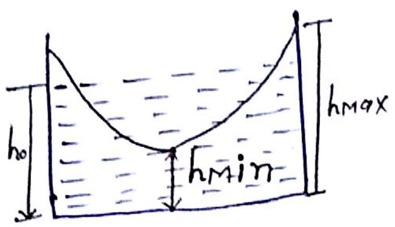


slope

$$\tan\theta = \frac{r\omega^2}{g} = \frac{dy}{dr}$$

$$y = \frac{r^2\omega^2}{2g} + \overset{*}{C}$$ ← const. of Integration.

Let initial height before Rotation is 'h₀'.



$$\boxed{h_0 = \frac{h_{min} + h_{max}}{2}}$$

at $r = 0$, $y = h_{min}$

$\therefore h_{min} = c$ — ①

at $r = R$, $y = h_{max}$

$$\boxed{h_{max} = \frac{R^2\omega^2}{2g} + \overset{*}{C}}$$ — ②

① + ②

$\Rightarrow h_{min} + h_{max} = \frac{R^2\omega^2}{2g} + 2c$

$\Rightarrow 2h_0 = \frac{R^2\omega^2}{2g} + 2c$

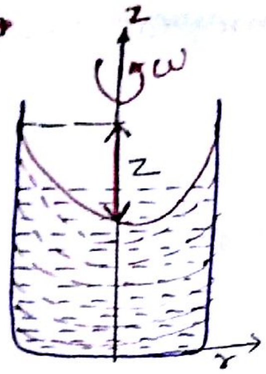
$\Rightarrow c = h_0 - \frac{R^2\omega^2}{4g}$

**

$$*h_{min} = h_0 - \frac{R^2\omega^2}{4g} \qquad *y = \frac{r^2\omega^2}{2g} + c$$

$$*h_{max} = h_0 + \frac{R^2\omega^2}{4g}$$

!! Do not use above formula If liquid is split out during Rotation.



→ $h = \frac{\omega^2 r^2}{2g} + C$

→ $h_{max} = h_0 + \frac{\omega^2 R^2}{4g}$ (at $r = R$)

→ $h_{min} = h_0 - \frac{\omega^2 R^2}{4g}$ (at $r = 0$)

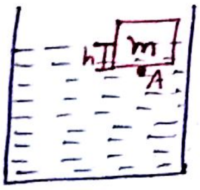
→ h_0 ⇒ Initial height of liquid before Rotation

$$Z = h_{max} - h_{min} = \frac{\omega^2 R^2}{2g}$$

Force of Bouancy (F_B) / Archemedes principle →

Vertical force applied by the liquid on solid when solid is partially or, fully immerged is called force of bouancy.

*Its Magnitude = weight of liquid displaced.



$F_B = mg$

$F_B = V_{in} \times \rho_L \times g$

$P_A = P_0 + \rho_L g h$

* Force of pressure on 'm'

$F_1 = P_0 A$ (↓)

$F_2 = P_A \cdot A$ (↑)

$F_{up} = F_2 - F_1$

$F_B = (P_A - P_0)A$

$F_B = \rho_L g h \cdot A$

$F_B = V_{in} \rho_L \cdot g$

→ Let, 'm' is fully submerged,

*$F_B = (P_{Bottom} - P_{top})A = (P_{top} + \rho_L g H - P_{top})A$

$F_B = (HA)\rho_L g$.

$$F_B = V_{in} \rho_L g$$

$$\text{Weight} = m_s g$$

$$m_s g = V_T \times \rho_s \times g$$

→ Body full float

If, $F_B = m_s g$

$V_{in} \rho_L = V_T \rho_s$

$$\frac{V_{in}}{V_T} = \frac{\rho_s}{\rho_s}$$

[V_T = Total volume
V_{in} = Submerged volume]

* → If $\boxed{\rho_s < \rho_L}$ ⇒ body will float with partially submerged.

$$\boxed{\frac{V_{in}}{V_T} = \frac{\rho_s}{\rho_L}}$$

* → If $\boxed{\rho_s = \rho_s}$ ⇒ body will float with fully submerged.

* → If $\boxed{\rho_s > \rho_L}$ ⇒ body will sink.

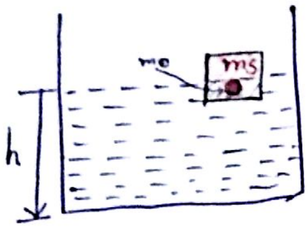
** → If body is fully submerged,

$$F_B = V_T \rho_L g$$
$$= V_T \rho_L g \times \frac{\rho_s}{\rho_s}$$

* $\boxed{F_B = \frac{\rho_L}{\rho_s} mg}$ * $\boxed{F_B \leq \frac{\rho_L}{\rho_s} mg}$

Rise & Fall of liquid level :→

Let the floating body is ice cube in which a coin of mass 'm_o' is present, What will be the effect on 'h' is ice melt.



→ Before ice melt (Initially)

$$F_B = V_1 \times \rho_L \times g$$

As body floats

$$F_B = (m_s + m_o) g$$
$$V_1 \rho_L g = (m_s + m_o) g$$
$$V_1 \rho_L = m_s + m_o \quad \text{—— (1)}$$

* $\boxed{V_1 = \frac{m_s + m_o}{\rho_L} = \frac{m_s + m_o}{\rho_w}}$

→ After ice melts,
volume of water formed $= V_2$

* $\boxed{V_2 = \frac{m_s}{\rho_s}}$

→ Volume of liquid displaced by coin $= V_3$

* $\boxed{V_3 = \frac{m_o}{\rho_c}}$

**
→ If $V_1 = V_2 + V_3$ ('h' will not change)
→ If $V_1 > V_2 + V_3$ ⇒ 'h' will decrese.
→ If $V_1 < V_2 + V_3$ ⇒ 'h' will increase.

* $$\boxed{\frac{m_s}{m_o} = \left(\frac{\rho_s}{\rho_c}\right)\left[\frac{\rho_c - \rho_L}{\rho_L - \rho_w}\right]}$$

→ If liquid is water

$$V_1 = \frac{m_s}{\rho_L} + \frac{m_0}{\rho_L} = \frac{m_s}{\rho_w} = \frac{m_0}{\rho_w}$$

$$V_2 + V_3 = \frac{m_s}{\rho_w} + \frac{m_0}{\rho_c}$$

$\because$ $\boxed{\rho_c > \rho_w}$

$\therefore$ $V_2 + V_3 < V_1$

$\Rightarrow$ **h will ↓**

→ If liquid is water & in place of coin a solid body is such that can float in water.

$\because$ $\boxed{\rho_c \le \rho_w}$

$$m_0 g = V_3 \rho_w g \qquad V_3 = \frac{m_0}{\rho_w}$$

$$*V_1 = \frac{m_s}{\rho_w} + \frac{m_0}{\rho_w}$$

$$*V_2 = \frac{m_s}{\rho_w}$$

$\because$ $\boxed{V_1 = V_2 + V_3}$

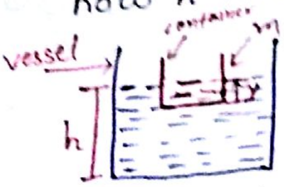
$\therefore$ Level will not change.

Exemplar III

→ A wooden cube is floating in water & a coin is placed over it. When coin fall in container how 'h' & 'y' will change.

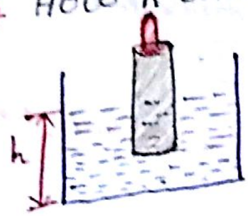
* Both will ↓ [As the coin fall into water, it displace some volume of water which is less, hence neglect the volume of coin].

→ 'm₀' gm of liquid is removed from vessel & droped in the container how 'h' will change.



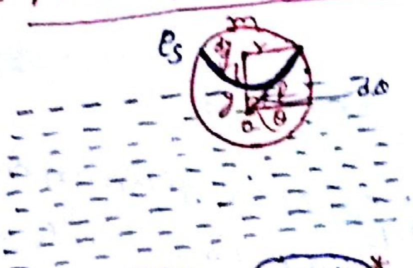
* 'h' remain same & y will ↑.

How 'h' will change when candle burn & wax melts.



* 'h' remain const.

Force of Bouancy when Sphere Floats →



→ If sphere floats

$$\boxed{F_B = mg}$$

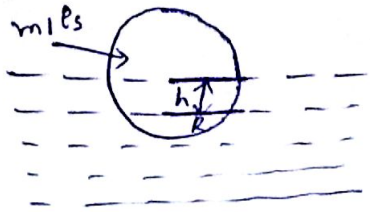
→ If it is not known that sphere float or not.

$$\boxed{F_B = V_{in} \times \rho_L \times g}$$

$$\boxed{V = \pi R^3 \left[\frac{2}{3} - \frac{h}{R} + \frac{h^3}{3R^3}\right]}$$

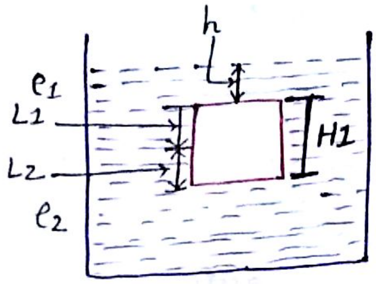
$$= \pi R^3 \left[-\cos\alpha + \frac{\cos^3\alpha}{3} + \frac{2}{3}\right]$$

$$\cos\alpha = \frac{h}{R}$$



$$F_B = \left(\frac{4}{3}\pi R^3 - \pi R^3\left[\frac{2}{3} - \frac{h}{R} + \frac{h^3}{3R^3}\right]\right)\rho_L g$$

Force of Bouancy when two liquid are mixed/Prescent.



$P_T = P_0 + \rho_1 g h$

✓ $P_B = P_0 + \rho_1 g (h + l_1) + \rho_2 g L_2$

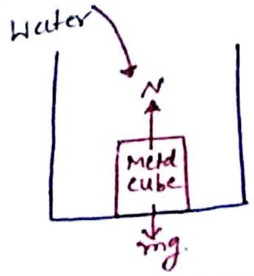
✓ $F_B = (P_B - P_T)A$

$= (\rho_1 g L_1 + \rho_2 g L_2)A$

* $F_B = (\rho_1 L_1 A) \times g + (\rho_2 L_2 A) \times g$

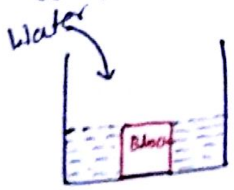
* $F_B = F_{B1} + F_{B2}$

** Water is droped at a const. Rate to fill the container, How force due to metal cube on the bottom of container changes with time.



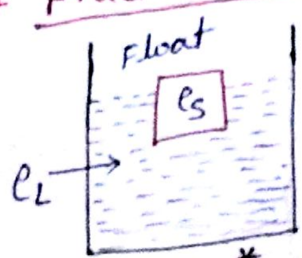
* Force due to cube will not change but due to water it will change.

** In above concept If water is filled so that block just submerged with its top edge open to atm, how normal Reaction b/w block & container will change. (assume $\rho_{block} < \rho_{water}$)



* 'N' Remain unchange.
[∵ Block is smooth ∴ Water can't enter b/w the surface of container & block].

Fraction of volume of solid inside liquid when solid is floating →

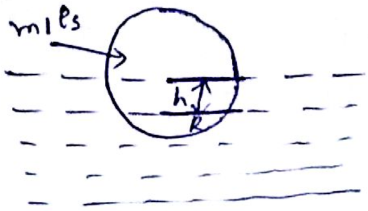


$F_B = mg$

$V_{in} \times \rho_L \times g = V \times \rho_s \times g$

$F_V = \frac{V_{in}}{V} = \frac{\rho_s}{\rho_L}$

** F_V does not depend on gravity [ie If we given an accn to the container in upward or, downward direction then F_V will remain same.]



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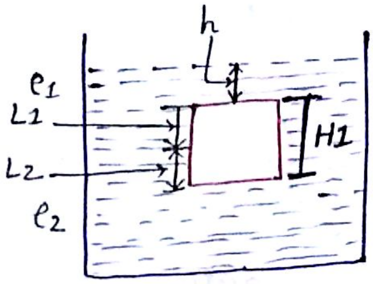

$P_T = P_0 + \rho_1 g h$

✓ $P_B = P_0 + \rho_1 g\,(h + l_1) + \rho_2 g L_2$

✓ $F_B = (P_B - P_T)A$

$= (\rho_1 g L_1 + \rho_2 g L_2)A$

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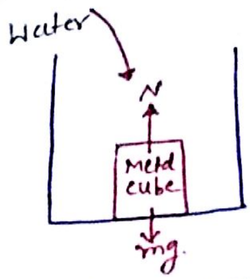

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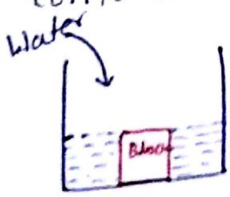

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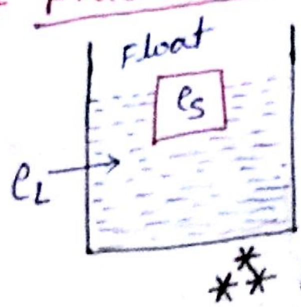
[∵ Block is smooth ∴ Water can't enter b/w the surface of container & block].

***

Fraction of volume of solid inside liquid when solid is floating→



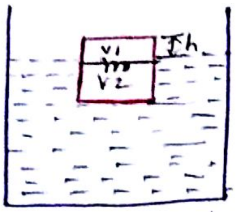
$F_B = mg$

$V_{in} \times \rho_L \times g = V \times \rho_s \times g$

$$F_V = \frac{V_{in}}{V} = \frac{\rho_s}{\rho_L}$$

** F_V does not depend on gravity [ie If we given an accl^n to the container in upward or, downward direction then F_V will remain same.]

**→ The system is placed on Earth & the container is open to atmosphere in this case 'h' is the height of block above water, how the value of 'h' will change is this system is placed on Moon. [assume water is not Evaporating]



*on Earth

$$\frac{V_1}{V_2} = \frac{\rho - \rho_2}{\rho_{air} - \rho}$$

$$= \boxed{\frac{\rho_2 - \rho}{\rho - \rho_{air}}}$$

*on Moon

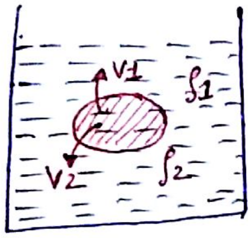
$$\frac{V_1'}{V_2'} = \frac{\rho_2 - \rho}{\rho}$$ [∵ Their is no air on Moon]

$$\because \frac{V_1}{V_2} > \frac{V_1'}{V_2'} \Rightarrow$$

∴ h will ↓

**→ When two immisible liquid of density ρ_1 & ρ_2 ($\rho_2 > \rho_1$) & Filled in a container. body of density ρ. ($\rho_1 < \rho < \rho_2$). ρ is Found to be Floating at interface of both liquid. If V1 vol. In liq ρ_1 & V2 volume of liquid ρ_2.

$$\because \frac{V_1}{V_2} = \frac{\rho_2 - \rho_1}{\rho_2 - \rho_1}$$



∴ Ratio of volume In upper & lower liq is

$$\boxed{\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}}$$

concept

* If $g_{eff} = 0$ then $F_B = 0$

→ Specific gravity (S.G) of a liq →

$$\frac{\text{density of liq } (\rho_l)}{\text{density of Water } (\rho_w)}$$

$$\boxed{\frac{\rho_l}{\rho_w} = \frac{V\rho_l g}{V\rho_w g}}$$ v = vol. of side

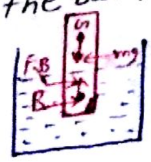
$$= \frac{\text{decrease in wt. in liq}}{\text{decrease in wt. in water}}$$

$$\Rightarrow \boxed{S.G = \frac{W_A - W_l}{W_A - W_w}}$$ (W_A: wt. in air, W_l: wt. in liq)

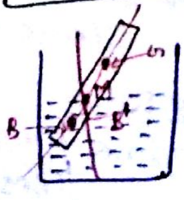
centre of the Bouancy (B) & the Metacentre (M)

'B' is the point from where 'F_B' (Force of bouancy) is assumed to acting. It is the centre of volume of liquid displaced.

* If the block is in equilibrium BG must be vertical.



$$\boxed{F_B = mg}$$



*When the body is rotated slightly from its equilibrium position it oscillate about Meta centre, If the equilibrium is Stable.

condition of equilibrium →

case-I → If the body is fully submerged in the liquid

(i) →

If the body is in equilibrium, then.

$F_B = mg$

← stable eqm.

(ii) →

$F_B = mg$
(stable)

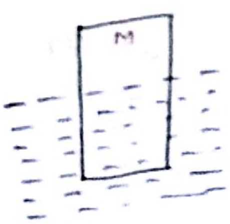
(unstable.)

Result (For fully submerged bodies in equilibrium).

(i) → If 'B' is above 'G' equilibrium is stable.
(ii) → If 'B' is below 'G' equilibrium is unstable.
(iii) → If 'B' & 'G' concides equilibrium is neutral.

case-II → Equilibrium of floating body.

* A body float in eqm. If $F_B = mg$



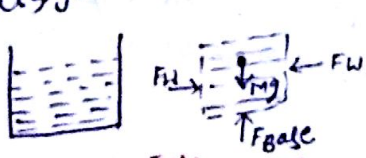
Result

* If 'M' is above 'G' eqm is stable.
* If 'M' is below 'G' eqm is unstable.
* If 'M' & 'G' concide eqm is neutral.

Force applied by fluid on a surface.

A liquid in a container apply a force on the base of container wich may be equal greater or, less than the weight of liq. in container.

case-I →



$F_{Base} = mg$ *

case-II →

$\Sigma F_V = 0$

$2F_W \sin\theta + F_{Base} = mg$ *

$F_{Base} < mg$

case III →

$\Sigma F_V = 0$

$2F \sin\theta + mg = F_{Base}$ *

$F_{Base} > mg$

Force applied by liquid on plance surface →

case-I → If the plance surface is horizontal →

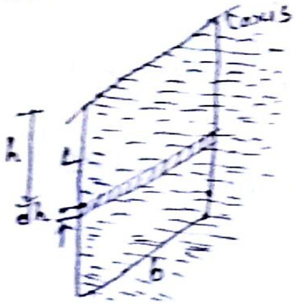
When the plane surface is horizontal & liq. has no horizontal accln. In this case pressure at all the points on surface is same ∴ force on the surface

$$\boxed{F = PA}$$

Where A = area of the surface.

This force will act from the centroid of surface.

case-II → Force on plane vertical surface →

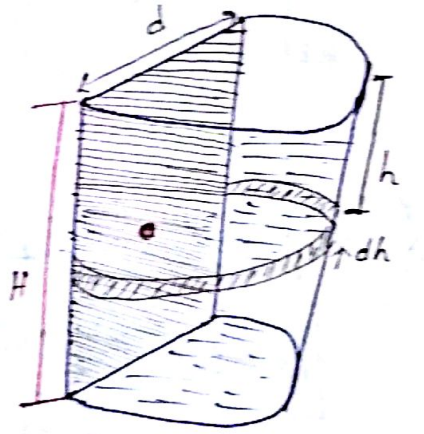


$$\boxed{F = P_{avg} \cdot Area \qquad P_{avg} = \rho g \frac{L}{2}}$$ → on vertical plane.

* Force applied by liquid on the curved surface of the cylinder!

→ As liquid inside cylinder is at rest hence net force on it is zero.

∴ Force applied by plane vertical surface = force applied by curved vertical surface.



✓ $F_{curved} = F_{plane}$

$= \rho g \frac{H}{2}(Hd)$

✓ $F_{curved} = \rho g \frac{H^2 d}{2}$

$$\boxed{F_{curved} = P_{avg} \times A_{pr}}$$

A_{pr} = Projected area.

$$\boxed{dF_{net} = 2\int dF \cos\theta}$$

$$\boxed{F_{ring} = 2\rho g R \cdot h \cdot dh}$$

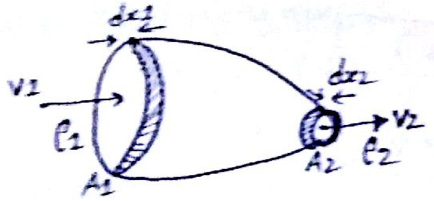
$$\boxed{F_{curved} = 2\rho g R \cdot \int_0^H h \cdot dh}$$

$$\boxed{F = \rho g R H^2}$$

2015 AIPMT ***

Equation of continuity

It is equation to conservation of mass.



$\rho_1 A_1 dx_1 = \rho_2 A_2 dx_2$

$$\boxed{\rho_1 A_1 v_1 = \rho_2 A_2 v_2}$$

* As liquid is ideal

∴ It is incompressible

∴ $\boxed{\rho_1 = \rho_2}$

$Av = const.$

* $\boxed{A_1 v_2 = A_2 v_2}$

$Av = const$

$A \frac{dx}{dt} = const$

$Q = \frac{dv}{dt} = const$

** $$\boxed{Q = \frac{d \cdot (volume)}{dt} = Av = const.}$$

↳ Rate of volume flow.

Bernoulli Principle →

It is equation to conservation of energy.

Forms of energy in the liquid flow →

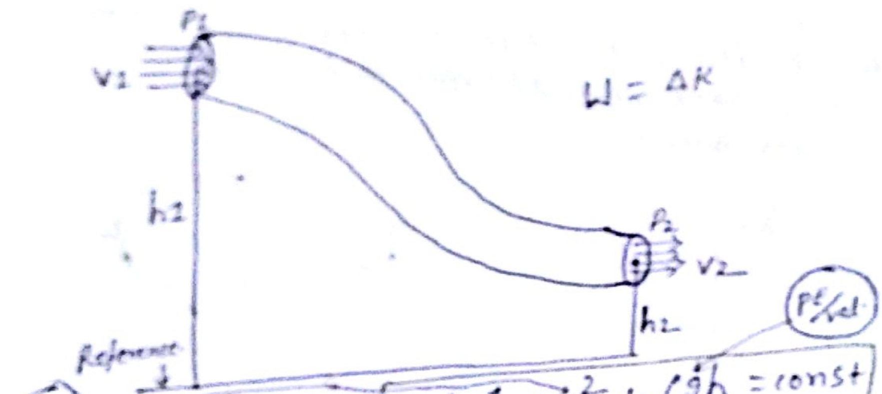
(1) Kinetic energy per unit volume →

$$K = \frac{K.E}{vol} = \frac{\frac{1}{2}(\Delta m)v^2}{\Delta V} = \frac{1}{2}\rho v^2$$

(2) → Potential energy per unit volume →

$$U = \frac{(\Delta m)gh}{\Delta V} = \rho gh$$

As we are considering ideal liquid/fluid which has no resistance in their motion.
∴ no heat loss & other losses will take place, hence energy is conserved.

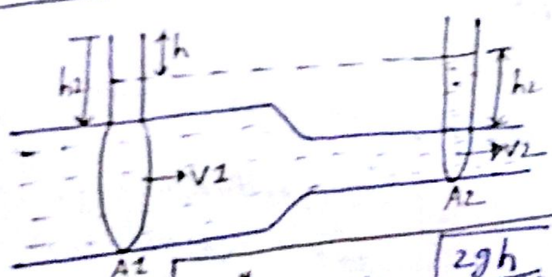


$$P + \frac{1}{2}\rho v^2 + \rho gh = const$$

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

Application of Bernoulli theorem

(1) → Venturimeter → use to determine flow of velocity.



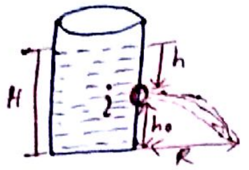
$$V_1 = A_2\sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

∴ Rate of Flow $= A_1 V_1$

$$= A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

|2| → Cylindrical container

|A| → Velocity of Efflux →



** $V = \sqrt{2gh}$

|B| → Range(R) →

$$R = v\sqrt{\frac{2h_0}{g}}$$

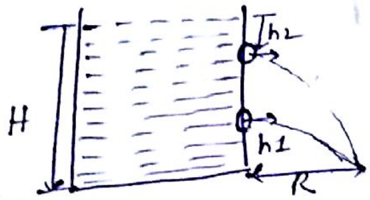
$$R = \sqrt{4hh_0}$$

* $R = \sqrt{4h(H-h)}$ * $R = 2\sqrt{h(H-h)}$

*For R = Max $\frac{dR}{dh} = 0 \Rightarrow \frac{4H-8h}{2\sqrt{4hH-4h^2}} = 0 \Rightarrow$ * $h = H/2$

* $R_{max} = H$

|C| →



$$R = \sqrt{4(H-h_1)h_1} = \sqrt{4h_2(H-h_2)}$$ *

$h_1 = h_2$

|D| → Force acting on the container due to efflux through a hole at depth 'h'

$$F = \rho . av^2$$

$F = \rho a \times 2gh$

* $F = 2a\rho gh$

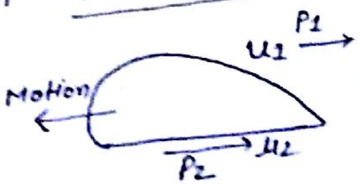
|E| → Height of free surface at any time 't'. If at t = 0, H = H₀. (assuming hole at the bottom of container i.e $h_0 = 0$)

$$\sqrt{\frac{2H_0}{g}} - \sqrt{\frac{2H}{g}} = \frac{a}{A}t$$

* Time after wich container become empty.

* $T = \frac{A}{a}\sqrt{\frac{2H_0}{g}}$

|3| → Dynamic lift in aeroplane →

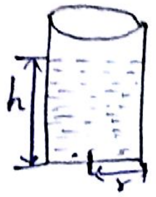


Let, A ⇒ Area of wing
m ⇒ mass of Aeroplane.

* Dynamic lift $F = (\Delta P)A = \frac{1}{2}\rho(v_1^2 - v_2^2)A$

* If t_1 is time after wich 1/2 liq. flow out & t_2 is time in wich remening half liq. flow out. Find $\frac{t_1}{t_2} \Rightarrow \frac{\sqrt{2}-1}{1}$ *

* Radius of a cyllindrical vessel is 'r' Find upto what height, liquid is filled 17 so that force on vertical wall & base is same?



$F_{wall} = P_{avg} \cdot Area$

$= \rho g \frac{H}{2} \times \rho g \frac{h}{2} \times 2\pi r h$

$F_{base} = \rho g h \times \pi r^2$

* $\boxed{h = r}$

Critical velocity & Reynold's Numer →

Max velocity up to wich flow Remain stedy are called critical velocity (v_c).

ρ → density of liq
v_c → critical velocity
D → diameter of pipe
η = cofficient of viscosity.

then,

$$\boxed{\text{Renold's no} = N_R = \frac{\rho v_c D}{\eta}}$$ ***

*If $N_R < 2000$ → Flow → steady.
* If $N_R > 2000$ → Flow → turbulent.

Types of liquid Flow

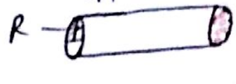
(a) → Streamline or, Steady Flow → At any point all the particle reaching at that point has same velocity as well as Follow the same path.

(b) → Turbulent Flow → velocity at any point varies with time.

2004 AIIMS
* In old age arteries carrying blood in Human body become narrow resulting in an ↑ in the B.P. This follow → Bernoulli principle.

* A cylindrical tube of spary pump has Radius 'R' one end of wich has 'n' Fine holes, each of Radius 'r'. Then the speed of liquid in the tube is 'V' the speed of ejection of the liquid througth hole.

VA = const



$\pi R^2 V = n \pi r^2 V_1$

* $\boxed{V_1 = \frac{VR^2}{n r^2}}$